Journal of Applied Mechanics and Technical Physics, Vol. 42, No. 6, pp. 979–985, 2001

TWO-FLUID HYDRODYNAMIC MODEL OF A BUBBLE FLOW

B. L. Kantsyrev[1] **and A. A. Ashbaev**[2]

UDC 532.54

A computational model for an unsteady one-dimensional gas–liquid flow taking into account gravity is proposed. The model includes the Zuber–Findlay relation and solutions of the Cauchy problem close to the solutions of drift models. It is shown that the effect of attached mass has a significant influence on the acoustic characteristics of the system of equations.

Introduction. Computational models for analysis of unsteady two-fluid flows are formulated as systems of differential equations of the mechanics of multiphase media [1]. There are one-fluid models with one differential equation of motion (for example, the hydraulic model used in the RELAP5 code [2]) and two-fluid models (used in the TRAC and CATHARE codes [3]).

An essential point in the development and further testing of one-fluid models is that empirical relations, such as Zuber–Findlay formulas [4] and flow pattern maps containing experimental data on gas–liquid flows, can be used as closing equations. This is important for modeling not only steady-state flows but also flows with kinematic wave processes. In more complex two-fluid models, the equations of motion for the carrier and disperse phases are written as two differential momentum equations. In the last case, the choice of parameters of the problem is more difficult. The equations of motion for heterogeneous flow components have not yet been finally formulated. The complexity of interaction between phases with different velocities complicates calculation of interphase forces, for example, the interphase friction force due to liquid viscosity and the coefficients of terms corresponding to collective interaction of disperse particles with the liquid flowing around them [5].

In the present paper, we study a system of equations of continuity and motion in which the coefficients mentioned above are considered as parameters that ensure satisfaction of empirical closing relations and specify the wave characteristics of the computational model. This system can be used to develop a two-fluid heat-hydraulic model which includes the equations of continuity, motion, and energy balance.

Main Assumptions. Let us write hydrodynamic equations of heterogeneous media for a gas–liquid monodisperse medium with barotropic components. Under the assumptions made, the closed system includes two momentum equations and two continuity equations. According to [1], the momentum equations have the form

$$\begin{aligned} \alpha_1\rho_1\Big(\frac{\partial V_1}{\partial t} + V_1\,\frac{\partial V_1}{\partial z}\Big) &= -\frac{\partial P}{\partial z} - F_{12} - F_{\rm w1} + \rho_1\alpha_1 g_z, \\ \alpha_2\rho_2\Big(\frac{\partial V_2}{\partial t} + V_2\,\frac{\partial V_2}{\partial z}\Big) &= F_{12} - F_{\rm w2} + \rho_2\alpha_2 g_z, \end{aligned} \qquad (1)$$

where the subscripts 1 and 2 refer to the liquid (carrier) and phase, respectively, the z axis is directed upward, in opposition to gravity, α_2 is the volume gas content, $\alpha_1 = 1 - \alpha_2$, V_2 and V_1 are the velocities of the bubble and liquid phases, respectively, $\partial P/\partial z$ are the forces due to the pressure gradient, $F_{\rm w1}$ and $F_{\rm w2}$ are the forces due to the interaction of flows with the channel wall, $\rho\alpha g_z$ is the gravity, and F_{12} is the interphase interaction force.

The quantity $F_{\rm w2}$ is considered small and is neglected. The interphase interaction force is

$$\begin{aligned} F_{12} = \rho_1\alpha_2\Big(\frac{\partial V_1}{\partial t} + V_1\,\frac{\partial V_1}{\partial z} - g_z\Big) - \rho_1\alpha_2\chi_m\Big(\frac{\partial V_2}{\partial t} + V_2\,\frac{\partial V_2}{\partial z} - \Big(\frac{\partial V_1}{\partial t} + V_1\,\frac{\partial V_1}{\partial z}\Big)\Big) \\ - \frac{n\pi a^2\rho_1}{2}\,C_\mu|U|U - k_\alpha\,\frac{\partial \alpha_2}{\partial z} - k_u\,\frac{\partial U}{\partial z}. \end{aligned}$$

---

[1]Russian Research Institute of Nuclear Plant Operation, Moscow 109507. [2]Osh Technological University, Osh 714000, Kirghizstan. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 42, No. 6, pp. 64–72, November–December, 2001. Original article submitted January 10, 2001.

Here the first term is the Archimedes force, the second term is the attached-mass force for spherical particles of constant radius in the liquid flow, the third term is the viscous friction force, and the forth and fifth terms are the forces due to the collective interaction between the liquid and the bubbles sliding relative to the liquid [5, 6], and χ_m is the coefficient that allows for the effect of nonsingularity and nonsphericity of disperse particles on the attached-mass force: $\chi_m = 1/2$ at $\alpha_2 \simeq 0$ and $\chi_m < 1/2$ at $\alpha_2 > 0$. If we set $\chi_m = 0$, the attached-mass force is not taken into account.

All bubbles are assumed to be of the same radius a; n is the bubble number density in the flow. The quantities a, n, and α_2 are related by $n = 3\alpha_2/(4\pi a^3)$. In this case, the interphase friction force becomes

$$F_{12} = \alpha_2 K_\mu |U| U.$$

Here $U = V_2 - V_1$ and $K_\mu = (3/8)(\rho_1/a)C_\mu^0 \psi_\alpha$, where C_μ^0 is the bubble resistance coefficient averaged over the channel cross section (for the reason of dimension, we can assume C_μ^0 to be a function of Reynolds and Laplace numbers, the ratio of densities ρ_1/ρ_2, and the ratio of the bubble size to the channel diameter a/D) and $\psi_\alpha(\alpha_2)$ is a dimensionless coefficient [1, Vol. 2] that allows for the effect of volume gas content on the interphase friction force.

We seek the coefficients k_α and k_u in the form

$$k_\alpha = R_\alpha(\rho_1, \rho_2, \alpha_2) U^2 n(4\pi a^3/3) = R_\alpha \alpha_2 U^2, \qquad k_u = R_u(\rho_1, \rho_2, \alpha_2) U n(4\pi a^3/3) = R_u \alpha_2 U,$$

where R_α and R_u have the dimension of density.

The form of the functions of C_μ^0, ψ_α, R_α, and R_u is further determined by comparison of the system of equations proposed with empirical relations.

System of Equations. In the absence of phase transitions, the continuity equations have the form

$$\frac{\partial(\rho_1\alpha_1)}{\partial t} + \frac{\partial(\rho_1\alpha_1 V_1)}{\partial z} = 0,$$
$$\frac{\partial(\rho_2\alpha_2)}{\partial t} + \frac{\partial(\rho_2\alpha_2 V_2)}{\partial z} = 0. \tag{2}$$

Solving Eqs. (1) for phase accelerations, we obtain

$$\frac{\partial V_1}{\partial t} + V_1 \frac{\partial V_1}{\partial z} = -\frac{\varphi_{1p}}{\rho_1}\frac{\partial P}{\partial z} - \varphi_{1\alpha} U^2 \frac{\partial \alpha_1}{\partial z} + \varphi_{1u} U \frac{\partial U}{\partial z}\frac{\alpha_2}{\rho_1}\frac{U|U|K_\mu}{1+\chi_m\rho/\rho_2} + g_z - F_{\text{w}1}\frac{1/\rho_1 + \chi_m/\rho_2}{1+\chi_m\rho/\rho_2}, \tag{3}$$

$$\frac{\partial V_2}{\partial t} + V_2 \frac{\partial V_2}{\partial z} = -\frac{\varphi_{2p}}{\rho_2}\frac{\partial P}{\partial z} - \varphi_{2\alpha} U^2 \frac{\partial \alpha_2}{\partial z} - \varphi_{2u} U \frac{\partial U}{\partial z} - \frac{\alpha_1}{\rho_2}\frac{U|U|K_\mu}{1+\chi_m\rho/\rho_2} + g_z - F_{\text{w}1}\frac{1/\rho_2 + \chi_m/\rho_2}{1+\chi_m\rho/\rho_2},$$

where $\varphi_{1p} = 1 + \alpha_2\varphi_p$, $\varphi_{2p} = 1 - \alpha_1\varphi_p$, $\varphi_p = (\rho_1 - \rho_2)(\chi_m/\rho_2)/(1 + \chi_m\rho/\rho_2)$, $\varphi_{1\alpha} = \varphi_\alpha\rho_2\alpha_2$, $\varphi_{2\alpha} = \varphi_\alpha\rho_1\alpha_1$, $\varphi_{1u} = \varphi_u\rho_2\alpha_2$, $\varphi_{2u} = \varphi_u\rho_1\alpha_1$, $\varphi_\alpha = R_\alpha/(\rho_1\rho_2(1+\chi_m\rho/\rho_2))$, $\varphi_u = R_u/(\rho_1\rho_2(1+\chi_m\rho/\rho_2))$, and $\rho = \rho_1\alpha_1 + \rho_2\alpha_2$.

System (2), (3) is closed. The characteristic equation corresponding to this system is

$$(V_1-\lambda)^2(V_2-\lambda)^2/(\rho_* C_*^2) - (V_1-\lambda)^2(\alpha_2/\rho_2)(\varphi_{2p} + \varphi_\alpha\alpha_1^2\rho_2(U/C_1)^2)$$
$$- (V_2-\lambda)^2(\alpha_1/\rho_1)(\varphi_{1p} + \varphi_\alpha\alpha_2^2\rho_1(U/C_2)^2) + \varphi_u U[\rho(V-\lambda)(V_1-\lambda)(V_2-\lambda)/(\rho_* C_*^2)$$
$$- (\alpha_1(V_2-\lambda) + \alpha_2(V_1-\lambda))] + \varphi_\alpha\alpha_1\alpha_2 U^2 = 0, \tag{4}$$

where $C_{1,2}$ are the speeds of sound in the first and second phases, C_* is determined from relations $1/\rho_* = \alpha_1/\rho_1 + \alpha_2/\rho_2$ and $1/(\rho_* C_*^2) = \alpha_1/(\rho_1 C_1^2) + \alpha_2/(\rho_2 C_2^2)$, and $V = (\rho_1\alpha_1 V_1 + \rho_2\alpha_2 V_2)/\rho$ is the weighted average velocity.

If $(U/C_1)^2 \ll 1$ and $(U/C_2)^2 \ll 1$ and terms of the corresponding and higher orders of smallness are ignored, Eq. (4) can be solved approximately

$$\lambda_{1,2} = (V_1 Y_1 + V_2 Y_2) \pm C_* K_{ac}, \quad \lambda_{3,4} = \rho_p\Big(V_2\frac{\alpha_1}{\rho_1}\varphi_{1p} + V_1\frac{\alpha_2}{\rho_2}\varphi_{2p} + \varphi_u\frac{U}{2}\Big) \pm U K_c, \tag{5}$$

where $Y_1 = \rho_p\alpha_1(\varphi_{1p}/\rho_1 + (\varphi_u/2)\alpha_2(\rho_1-\rho_2)Q/\rho_p)$, $Y_2 = \rho_p\alpha_2(\varphi_{2p}/\rho_2 - (\varphi_u/2)\alpha_1(\rho_1-\rho_2)Q/\rho_p)$, $K_{ac} = (\rho_*/\rho_p)^{0.5}$, $K_c = \rho_p[-\alpha_1\alpha_2\varphi_{1p}\varphi_{2p}/(\rho_1\rho_2) + \alpha_1\alpha_2\varphi_\alpha/\rho_p + \varphi_u^2/4 - Q\alpha_1\alpha_2\varphi_u/\rho_p)]^{0.5}$, $1/\rho_p = (\alpha_1/\rho_1)\varphi_{1p} + (\alpha_2/\rho_2)\varphi_{2p}$, and $Q = (\rho_1-\rho_2)\rho_p/(\rho_1\rho_2(1+\chi_m\rho/\rho_2))$. The characteristics $\lambda_{1,2}$ correspond to propagation of acoustic perturbations in the two-phase flow, and characteristic $\lambda_{3,4}$ correspond to propagation of convective perturbations.

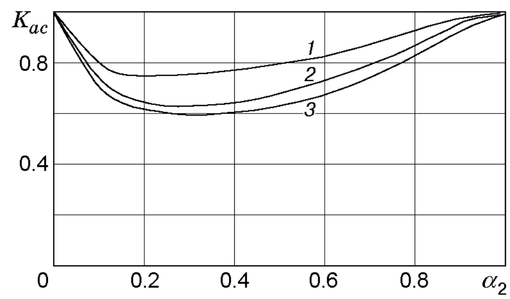

Fig. 1. K_{ac} versus α_2 for $\chi_m = 0.2$ (curve 1), 0.5 (curve 2), and 0.8 (curve 3).

It follows from (5) that allowance for the effect of attached mass influences the acoustic properties of system (2), (3). Figure 1 shows a curve of the coefficient K_{ac} versus the volume gas content at $\rho_2/\rho_1 = 0.05$ for three values of χ_m. As can be seen from Fig. 1, the attached-mass effect results in a decrease in the speed of sound compared with the value of C_*. Stadtke et al. [7] allowed for the effect of attached mass on the speed of sound in a water flow with bubbles of constant sizes [7] and obtained a relation for the speed of sound that coincides with the data given in the present paper if terms of the order of smallness $(U/C_*)^2$ and higher are ignored. Calculations of the critical discharge of boiling water from a rectangular channel into the atmosphere [7] and comparison with the experiment of [8] led Stadtke et al. [7] the conclusion that allowance for the attached-mass force is important for modeling critical discharges.

To analyze the convective properties of system (2), (3) regardless of its acoustic properties, we consider gas–liquid flow regimes in which the phase compressibility is insignificant ($\rho_1 = \text{const}$ and $\rho_2 = \text{const}$ as $C_{1,2} \to \infty$). In this case, the total volume flow can be treated as a boundary condition and a specified function of time. Then, Eqs. (2) leads to the equations of volume flow W and volume gas content α_2:

$$\frac{\partial W}{\partial z} = 0, \qquad W(t) = \alpha_1 V_1 + \alpha_2 V_2; \tag{6}$$

$$\frac{\partial \alpha_2}{\partial t} + \alpha_1\alpha_2 \frac{\partial U}{\partial z} + [W + U(\alpha_1 - \alpha_2)]\frac{\partial \alpha_2}{\partial z} = 0. \tag{7}$$

Excluding the derivative of the pressure with respect to the coordinate from Eq. (3) and allowing for the independence of the volume flow W from the coordinate, we obtain the differential equation

$$\frac{\partial U}{\partial t} + f_u \frac{\partial U}{\partial z} + f_\alpha \frac{\partial \alpha_2}{\partial z} = f, \tag{8}$$

where $f_u = W + U[\alpha_1 - \alpha_2 - 2Q\alpha_1\alpha_2 + QR_u/(\rho_1 - \rho_2)]$, $f_\alpha = U^2[R_\alpha - \rho(1 + \chi_m\rho_1/\rho)]Q/(\rho_1 - \rho_2)$, $f = Q[(-g_z + dW/dt) - K_\mu U|U|/(\rho_1 - \rho_2)]$, and $Q = (\rho_1 - \rho_2)\rho_p/(\rho_1\rho_2(1 + \chi_m\rho/\rho_2))$.

The right side of Eq. (8) does not contain the force of friction against the channel wall F_{w1} that acts on the first phase, which was previously noted in [9]. If the flow is almost homogeneous and steady, the left side of Eq. (8) is much less than every term on its right side and F_{w1} because of the smallness of the derivatives $\partial U/\partial t$, $\partial U/\partial z$, and $\partial\alpha_2/\partial z$, and $|dW/dt| \ll g$.

If $f = 0$, then

$$|g_z|(\rho_1 - \rho_2) = K_\mu U|U|, \tag{9}$$

i.e., $U = U(\alpha_2)$. Substituting $U = U(\alpha_2)$ into Eq. (7), we obtain the equation of the drift model:

$$\frac{\partial \alpha_2}{\partial t} + V_\alpha \frac{\partial \alpha_2}{\partial z} = 0. \tag{10}$$

Here $V_\alpha = W + d(U(\alpha_2)\alpha_1\alpha_2)/d\alpha_2$.

To obtain a convenient form of the empirical relation that provides for additional information on the coefficients C_μ^0, ψ_α, R_α, and R_u, we consider the establishment of a gas–liquid countercurrent flow in a vertical channel. We assume that the upper part of the vertical channel is connected with a large container filled with a liquid and its lower part is connected with a similar container filled with a gas. Obviously, moving down, the liquid flow is limited by the ascending gas countercurrent flow. In this case, the total volume flow in the channel will be equal

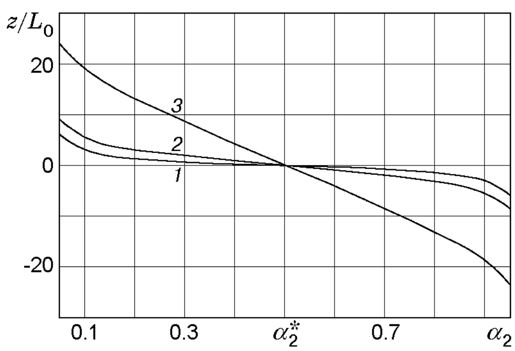

Fig. 2

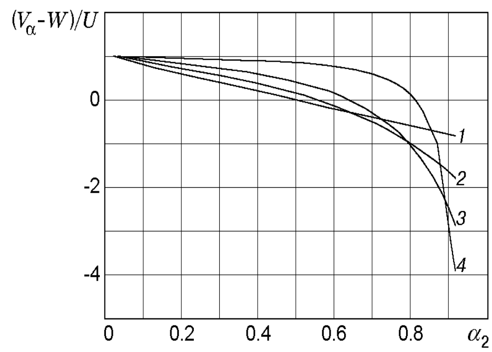

Fig. 3

Fig. 2. Volume gas content α_2 versus the coordinate z/L_0 for $t/(L_0/U) = 0$ (curve 1), 3 (curve 2), and 20 (curve 3).

Fig. 3. $(V_\alpha - W)/U$ versus α_2 for $\rho_2/\rho_1 = 1$ (curve 1), 0.2 (curve 2), 0.05 (curve 3), and 0.001 (curve 4).

to zero. (This example is not an abstract one. Indeed, at the final stage of a hypothetical failure of a coolant in a water-cooled reactor, water boiling can result in rapid growth in vapor content, and, consequently, unstable "hanging" of a layer of underheated water on a vapor "cushion" can give rise to a countercurrent flow corresponding to the present model.) Calculations of countercurrent flow for the drift flow model [10] use the drift flow relation

$$W_{21} = U_0 \alpha_2 \alpha_1^N, \tag{11}$$

where $N = 0$–2 and U_0 is the floating velocity of a single bubble in an unbounded liquid volume. At the same time, by the definition of the drift flow,

$$W_{21} = W_2 - \alpha_2 W. \tag{12}$$

Simultaneous solution of Eqs. (11) and (12) yields the steady-state value of α_2 for the specified reduced gas flow rate W_2 and volume flow W. In this case, $W = 0$. Let $N = 1$, which corresponds to $U = V_2 - V_1 = U_0$. Then, for small values of W_2, the joint solution of (11) and (12) with respect to α_2 is a solution of a quadratic equation whose one root is larger than 0.5 and the other is smaller than 0.5. To solve this problem uniquely, we need some additional information. Let us consider the solution of (10) simultaneously with the condition $U = U_0 = \text{const}$.

Figure 2 shows distributions of the volume gas content in the channel obtained from the solution of (10) for various times [$L_0 = a(8/3)(\rho/\rho_1)/(C_\mu^0 \psi_\alpha)$ is the characteristic scale of change in α_2 at the initial moment]. At the initial moment, the volume gas content in the channel corresponds to curve 1. It can be seen that the gas content profile flattens gradually with time, and a steady-state flow with uniform distributions of velocities and gas content is established. The establishment process occurs in accordance with the wave properties (10). Indeed, for values of α_2 approaching zero (upper part of the channel), $V_\alpha > 0$ and for values of α_2 approaching unity (lower part of the channel), $V_\alpha < 0$ ($V_\alpha = 0$ for $\alpha_2 = \alpha_2^* = 0.5$). Curve 1 in Fig. 3 corresponds to the dependence of $(V_\alpha - W)/U_0$ on α_2 at $U = \text{const}$.

As follows from Figs. 2 and 3, the upward transfer of convective perturbations corresponds to $\alpha_2 < \alpha_2^*$ and the downward transfer corresponds to $\alpha_2 > \alpha_2^*$. As a result, a flow with the parameters

$$\alpha_2 = \alpha_2^*, \qquad V_\alpha = 0 \tag{13}$$

is established. Using the condition $U = U_0 = \text{const}$ and relation (13) for volume rates of the steady-state descending and ascending flows ($W_1 = V_1\alpha_1$ and $W_2 = V_2\alpha_2$), we obtain the relation

$$W_1^{1/2} + W_2^{1/2} = U_0^{1/2}, \tag{14}$$

which corresponds to the flow choking regime at $U = \text{const}$. The structure of relation (14) coincides with the structure of the well-known Wallis relation [10] for countercurrent flow "choking." At $W = 0$, there is a unique value $\alpha_2 = 0.5$. In this example, it is impossible to obtain a unique solution of the problem of countercurrent flow establishment using only the specified boundary conditions and equations of conservation of mass and momentum balance in stationary form. Indeed, from Eq. (13) and the condition $dV_\alpha/d\alpha_2 < 0$, we obtain the condition of flow choking in the channel relative to convective perturbations that arrive at the channel from its boundaries and find that the steady-state solution is independent of the boundary conditions (if $\alpha_2 > \alpha_2^*$ in the lower part of the channel and $\alpha_2 < \alpha_2^*$ in the upper part). In the indicated approach, described in [11], the countercurrent flow choking regime can be regarded as a kinematic analog of the critical outflow and the corresponding empirical relations can be treated as data on the convective characteristics of the system of equations for two-velocity flows (similarly, data on the critical mass flows are used to specify acoustic characteristics). Therefore, it is appropriate to use the Wallis formula [10] for the choking regime as a closing relation:

$$W_1^{0.5} + (\rho_2/\rho_1)^{0.25} W_2^{0.5} = C^{0.5}. \tag{15}$$

Here $C^{0.5} = C_f(gD(\rho_1 - \rho_2)/\rho_1)^{0.25}$ and, according to [10], C_f can be considered a function of the dimensionless number $N_f = \mathrm{Re}(\rho_1 - \rho_2)/\rho_1$, where $\mathrm{Re} = [(gD)^{0.5} D\rho_1]/\mu_1$ is the Reynolds number.

From (13) and (15), we obtain the relation for sliding

$$U = C/(\alpha_1 + (\rho_2/\rho_1)^{0.5}\alpha_2). \tag{16}$$

Relation (16) corresponds to the Zuber–Findlay equation [4] for phase sliding, which is a generalization of experimental data for flows in vertical channels:

$$V_2 = C_0 W + V_w. \tag{17}$$

From (17) and the equality $W = \alpha_1 V_1 + \alpha_2 V_2$ it follows that $U = ((C_0 - 1)W + V_w)/\alpha_1$ if $C_0 = 1$ and $V_w = C/(1 + (\rho_2/\rho_1)^{0.5}\alpha_2/\alpha_1)$.

An analysis of the experimental data of [10, 12–14] shows that linear dependences of the form (17) are valid over a wide range of countercurrent flow parameters. Hence, relation (15), obtained in [10] for annular flow, can be used in determining parameters of the present hydrodynamic model. Relation (15) is used to analyze experimental data on "choking" in channels where water is supplied through porous inserts (see [10]) to minimize the effect of boundary conditions. This suggests that relation (15) is determined only by flow parameters in the channel and not at its edges, and, hence, can be used to determine parameters of the one-dimensional model. From comparison of (16) with (9), it follows that $C_\mu^0 = (8/3)a/[DC_f(\mathrm{Re}, \rho_2/\rho_1)]$ and $\psi_\alpha = (\alpha_1 + (\rho_2/\rho_1)^{0.5}\alpha_2)^2$.

Within the framework of the model proposed, to ensure satisfaction of the flow choking conditions relative to convective perturbations that come to the channel from its boundaries when the countercurrent flow choking regime is established, we require that the characteristic equation of system (6)–(8) have degenerate roots whose values are equal to V_α in Eq. (10). This condition is satisfied if

$$R_\alpha = \rho(1 + \chi_m\rho_1/\rho) - \alpha_1\alpha_2(\rho_1 - \rho_2)\delta_\alpha^2/Q, \qquad R_u = 2\alpha_1\alpha_2(\rho_1 - \rho_2)(1 + \delta_\alpha/Q), \tag{18}$$

where $\delta_\alpha = (1 - (\rho_2/\rho_1)^{0.5})/\psi_\alpha^{0.5}$.

Thus, the parameters of the hydrodynamic model are defined. To compare the relations obtained for R_α and R_u with the results of [6], we write the equation following from (18) for the interphase force acting on a particle in the liquid flow providing that $\rho_2/\rho_1 \ll 1$ and $\alpha_2 \ll 1$ and ignoring the friction force:

$$F_{12} = 4.3\pi a^3\rho_1\Big\{\frac{\partial V_1}{\partial t} + V_1\frac{\partial V_1}{\partial z} - g_z - \frac{1}{2}\Big[\frac{\partial V_2}{\partial t} + V_2\frac{\partial V_2}{\partial z} - \Big(\frac{\partial V_1}{\partial t} + V_1\frac{\partial V_1}{\partial_z}\Big)\Big] - K_1U^2\frac{\partial \alpha_2}{\partial z} - K_2\alpha_2 U\frac{\partial U}{\partial z}\Big\}.$$

In the present paper, $K_1 = 1.5$ and $K_2 = 3$. In [6], $K_1 = 0.6$ and $K_2 = 0.9$. The difference in the values of K_1 and K_2 is explained by the fact that Kroshilin and Kroshilin [6] calculated the interphase force assuming that the distribution of disperse particles in the flow was chaotic, while in the present paper, we used empirical data obtained for real flows. We note that the value of K_1 should not be less than 1.5 in order for the characteristic equation of system (2)–(3) to have only real roots. If (18) is satisfied, system (6)–(8) has the characteristics $\lambda_1 = V_\alpha$ and $\lambda_2 = V_\alpha$. Substituting (18) into system (2)–(3), we can show that the convective characteristics of system (2), (3) are also degenerate and equal to V_α. Since the systems of equations considered above have degenerate characteristics, the correctness of the formulation of the Cauchy problem requires an additional analysis. Because the stability of solutions of the Cauchy problem for system (2), (3) is rather difficult to estimate, we estimate the stability for the simpler system (6)–(8), which follows from (2) and (3) at $\rho_1 = \mathrm{const}$ and $\rho_2 = \mathrm{const}$. Let us consider the

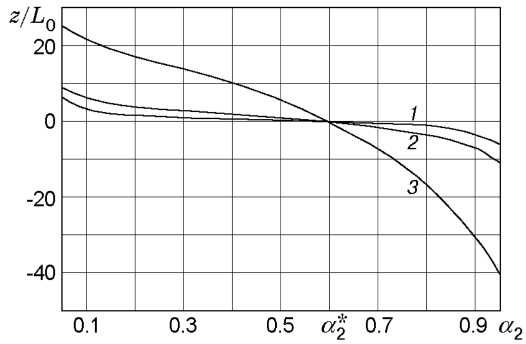

Fig. 4. α_2 versus z/L_0 for $t/(L_0/U) = 0$ (curve 1), 3 (curve 2), and 20 (curve 3).

development of weak perturbations of a homogeneous steady state that is a homogeneous steady-state solution of two equations of motion [1, Vol. 1]. After linearization of Eqs. (6)–(8) with respect to this homogeneous state, we seek the condition of existence of a nontrivial solution having the form of a traveling wave: $U = U_0 + U' \exp(i\omega t - ikz)$ and $\alpha_2 = \alpha_{20} + \alpha_2' \exp(i\omega t - ikz)$. This condition is satisfied if the determinant of the linearized system of equations vanishes [with allowance for the linearized right side (8)]. Reducing the determinant to zero, we have $i\omega_1 = iV_\alpha k + Q\partial(f/Q)/\partial U$ and $\omega_2 = V_\alpha k$, where $\partial(f/Q)/\partial U < 0$.

Thus, there are two solutions of the dispersion equation. One solution corresponds to a damping wave and the other is a neutral-stable solution, as in the drift equation. It is important that the phase wave velocities are equal to V_α, i.e., the choking condition for the steady-state countercurrent flow is satisfied for convective perturbations of all frequencies. In other words, all small perturbations of the phase sliding and gas content U' and α_2' will be carried out of the channel to its boundaries. This leads to establishment of a channel countercurrent flow with the parameters corresponding to (15). The result obtained shows that the formulation of the Cauchy problem for system (6)–(8) is correct.

Calculation Results. To illustrate the kinematic characteristics of the hydrodynamic model described by Eqs. (2) and (3), we solved numerically the problem of establishment of a countercurrent flow regime in a vertical channel within the framework of the indicated system of equations. Numerical integration was performed by the method of characteristics. We first determined the relations for converting the variables into the next time layer for the nondegenerate system of equations. Next, passing to the limit, we obtain degeneration of the convective characteristic). The calculation domain is similar to that described above. Obviously, for the adopted geometry of the calculation domain, the total volume flow is zero. This regime was chosen because with zero volume flow, the steady-state value of the volume gas content for the drift model depends only on the parameter ρ_2/ρ_1, defined in (15) and, hence, one does not need to model the flow circuit in which the specified volume flow is maintained. It follows from Fig. 3 that the regime with $W = 0$ corresponds to steady-state values $\alpha_2 > 0.5$. The value $\rho_2/\rho_1 = 0.2$ chosen in the calculations corresponds to a steady-state regime with $\alpha_2^* = 0.6$.

Figure 4 shows distributions of the volume gas content in the channel for various times. It can be seen that the establishment process leads to a steady-state regime with a steady-state value of α_2^* equal to 0.6. Thus, the convective wave characteristics of this model correspond to the empirical relation (15). Choking of kinematic perturbations at the upper and lower boundaries of the vertical channel implies that the calculation result does not depend on the volume gas content in the upper and lower containers if the gas content in the upper container is less than α_2^* and in the lower container, it is larger than α_2^*. Thus, the calculations of the establishment process using the bubble flow model is correct even if the gas content in the lower container is close to unity.

Conclusions. A two-fluid hydrodynamic model for a disperse flow was proposed. The choice of parameters of this model allows for the Zuber–Findlay relation as a closing equation. The analysis performed shows that the countercurrent flow choking regime in a vertical channel can be considered as a two-velocity flow choked relative to external convective perturbations and the choking condition can be used to chose parameters of the problem. Allowance for the effect of attached mass in the model of a two-fluid disperse flow with bubbles of fixed sizes results in a decrease in the estimated speed of sound [coefficient $(\rho_*/\rho_p)^{0.5}$ is always smaller than unity], which is important for calculations of critical outflows.

REFERENCES

1. R. I. Nigmatulin, *Dynamics of Multiphase Media*, Hemisphere, New York (1991).
2. V. H. Ransom, *RELAP5/MOD1 Code Manual*, Vol 1: *Code Structure, System Models, and Numerical Method*, NUREG/CR-1826 (1982).
3. F. Barre and M. Bernard, "The CATHARE code strategy and assessment," *Nuclear Eng. Design*, **124**, 257–284 (1990).
4. N. Zuber and J. A. Findlay, "Average volumetric concentration in two-phase flow systems," *J. Heat Transfer*, No. 12, 453–540 (1965).
5. D. Drew, L. Cheng, and R. T. Lahey, "The analysis of virtual mass effects in two-phase flow," *Int. J. Multiphase Flow*, **5**, No. 4, 233–242 (1979).
6. A. E. Kroshilin and V. E. Kroshilin, "Calculation of the interphase force in a monodisperse bubble medium with an ideal carrier phase and chaotic distribution of inclusions," *Prikl. Mekh. Tekh. Fiz.*, No. 5, 95–103 (1986).
7. H. Stadtke, G. Franchello, and B. Worth, "Numerical simulation of multi-dimensional two-phase flow based on hyperbolic flow equations," in: *Proc. 31st Meeting of Europ. Two-phase Flow Group* (Piacenza, Italy, June 6–8, 1994), Part 1 (1994), pp. 783–798.
8. A. R. Edwards and T. P. O'Brien, "Studies of phenomena connected with the depressurization of water reactors," *J. Brit. Nuclear Eng. Soc.*, **9**, No. 2, 125–135 (1970).
9. R. I. Nigmatulin, "Drift flux model, as approximation of two fluid model for two-phase dispersed and slug flow in tube," in: *Scholarship at Rensselaer Polytecnic. Institute* (U.S.A., 1994), S.l. (1994), pp. 141–163.
10. G. Wallis, *One-Dimensional Two-Phase Flow*, McGraw-Hill, New York (1969).
11. B. L. Kantsyrev and R. I. Nigmatulin, "Countercurrent flow in pressurized light water reactor," in: *Proc. 5th Int. Topical Meeting on Reactor Thermal Hydraulics (NURETH-5)* (Salt Lake City, U.S.A., Sept. 21–24, 1992), S.l., **5** (1992), pp. 1473–1474.
12. M. Sobajima, "Experimental modeling of steam-water countercurrent flow limit for perforated plates," *J. Nuclear Sci. Technol.*, **22**, 723–732 (1986).
13. Y. Taitel and D. Barnea, "Countercurrent gas–liquid vertical flow, model for flow pattern and pressure drop," *Int. J. Multiphase Flow*, **9**, No. 6, 637–647 (1983).
14. S. M. Ghiaasiaan, "Countercurrent two-phase flow regimes and void fraction in vertical and inclined channels," *Nuclear Sci. Eng.*, **119**, No. 3, 182–187 (1995).